



# Individual tree diameter increment model for managed even-aged stands of ponderosa pine throughout the western United States using a multilevel linear mixed effects model<sup>☆</sup>

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## ABSTRACT

A diameter increment model is developed and evaluated for individual trees of ponderosa pine throughout the species range in the United States using a multilevel linear mixed model. Stochastic variability is broken down among period, locale, plot, tree and within-tree components. Covariates acting at tree and stand level, as breast height diameter, density, site index, and competition indices are included in the model as fixed effects in order to explain residual variability. The data set used in this study came from long-term permanent research plots in even-aged, pure stands both planted and of natural origin. The data base consists of six levels-of-growing stock studies supplemented by initial spacing and other permanent-plot thinning studies for a total of 310 plots, 34,263 trees and 153,854 observations. Regression analysis is the preferred technique used in growth and yield modeling in forestry. We choose the mixed effects models instead of the regression analysis approach because it allows for proper treatment of error terms in a repeated measures analysis framework. Regional growth and yield models exist for ponderosa pine. However, data collection and analysis procedures differ. As a result, comparisons of growth responses that may be due to geographic variation of the species are not possible. Our goal is to present a single distance-independent diameter increment model applicable throughout the geographic range of ponderosa pine in the United States and by using only data from long-term permanent plots on sites capable of the productivity estimated by Meyer [Meyer, W.H., 1938. Yield of Even-Aged Stands of Ponderosa Pine. US Department of Agriculture Technical Bulletin 630].

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## 1. Introduction

Ponderosa pine (*Pinus ponderosa* Dougl.) is one of the most widely distributed conifers in North America. It occurs in 15 western states, extending from the western Great Plains to the Pacific Coast and from southern British Columbia, Canada, to Baja California, Mexico. It occurs in pure stands or in association with sugar pine (*Pinus lambertiana* Dougl.), incense-cedar (*Libocedrus decurrens* Torr.), Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco), Jeffrey pine (*Pinus jeffreyi* Grev. & Balf.), limber pine (*Pinus flexilis* James), oaks (*Quercus* sp.), junipers (*Juniperus* sp.), and true fir (*Abies* sp.). Ponderosa pine is found at elevations ranging from sea level in the

northern part of its range to 10,000 feet in the southwestern United States (Oliver and Ryker, 1990). Throughout this vast area ponderosa pine is one of the most valued tree species. Recognized initially for its wood quality and as a major source of forage for cattle, ponderosa pine forests are now recognized as vital wildlife habitat, and they provide abundant recreational opportunities. As a result, ponderosa pine forests have a long history of intensive management.

Growth and yield models and tree diameter growth models in particular are invaluable tool for forest management planning at any level. Accurate estimates of both current resource levels and the expected resource changes from implementing various management alternatives are needed for making wise management decisions. In these models, diameter growth is expressed as a function of tree size and vigor effects, competition effects, and site effects (Cole and Stage, 1972; Dolph, 1988; Wykoff, 1990; Dolph, 1992; Hann and Hanus, 2002; Mailly et al., 2003; Calama and Montero, 2004; Trasobares and Pukkala, 2004; Zhao et al., 2004; Calama and Montero, 2005).

Regression analysis is the most commonly used statistical method in forest modeling (Gregoire et al., 1995). The data set for

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these research studies come from data measured repeatedly over time on the same tree (multiple observations obtained from the same sampling unit or subject in sequence over time), also known as longitudinal data. Without question, research studies with repeated measure designs are fundamental to most ecological and biological research (Gutzwiller and Riffell, 2007). However, the nature of repeated measure design and these hierarchical structures are often ignored and independence between observations is assumed (Biging, 1985; Lappi, 1986; Searle et al., 1992; Gregoire et al., 1995; Keselman et al., 1999; Littell et al., 2000; Kowalchuk and Keselman, 2001; Garrett et al., 2004; Calama and Montero, 2005; Gutzwiller and Riffell, 2007).

Growth and yield researchers preferring Ordinary Least Squares Estimation (OLSE) technique to fit a regression model to repeated measures data argue that the ordinary least squares estimates are unbiased; and that more reasonable variance structure may not be necessary since users are mainly interested in prediction (Monleon, 2004). This argument is not supported by the statistical model used because it violates the first fundamental assumption needed to apply the OLSE method. It violates the assumption of independent observations. The fact is that since the same experimental unit is measured several times makes the re-measured observations correlated (Garrett et al., 2004; Hanke and Wichern, 2005; Gutzwiller and Riffell, 2007). Models that ignore the plot (sampling unit) effect and the error structure imposed by the sampling scheme assume that the relationship between tree growth and predictor variables within-plot is the same as that between-plot (Gregoire et al., 1995; Monleon, 2004; Garrett et al., 2004; Calama and Montero, 2005).

For repeated measures data, the sample of observations cannot be regarded as a random sample. Further, strong autocorrelation can make two mutually exclusive variables appear to be related. Consequently, application of regression analysis techniques to observations of this kind can produce a significant regression. This type of estimated relationship is said to be spurious. Spurious regression problems are detected by the examination of the plot of the residuals against time. When regression analysis techniques are applied to time dependent data, spurious regression may go undetected with a serious misinterpretation of the results (Hanke and Wichern, 2005).

Additionally, mixed models represent a significant improvement over traditional repeated measures analysis using regression analysis approach because regression analysis technique does not readily allow for missing data. For example, if an observation for one individual is missing for one of the time periods, the data for all time periods for that individual must be excluded from the analysis, unless an estimate for the missing datum can be generated. Sometimes it is reasonable to do this by computing a mean based on the other observations in the same treatment group and time period, but this approach reduces the variance of the group and thereby alter the outcome of the analysis in ways that are not defensible. Mixed models on the other hand, accommodate incomplete records without the need for such estimate (Littell et al., 1996; Gutzwiller and Riffell, 2007).

The analytical techniques for assessing statistical significance must be appropriate for the error structure imposed by the sampling scheme otherwise the model mean square error may underestimate the variance of the coefficient estimators and also produce incorrect estimates for the confidence interval of the parameters (Biging, 1985; Gregoire et al., 1995; Keselman et al., 1999; Littell et al., 2000; Kowalchuk and Keselman, 2001; Garrett et al., 2004; Calama and Montero, 2005; Hanke and Wichern, 2005; Gutzwiller and Riffell, 2007). A fitting routine that does not account for repeated measures in the model will produce unreliable

confidence intervals for prediction (Garrett et al., 2004; Hanke and Wichern, 2005; Gutzwiller and Riffell, 2007). The more complicated error structure of this type of data has often been ignored in forestry, with some exceptions (Gregoire et al., 1995). To overcome this problem, growth and yield researchers have widely proposed multilevel linear and nonlinear mixed models with both fixed and random components (Biging, 1985; Gregoire et al., 1995; Hall and Bailey, 2000; Calama and Montero, 2005; Lynch et al., 2005; Uzoh and Oliver, 2006; Gutzwiller and Riffell, 2007).

Mixed model calibration of tree growth increment is based on the fact that the stochastic component of growth variability is a consequence of different factors acting simultaneously (Calama and Montero, 2005; Gutzwiller and Riffell, 2007). If it is considered that the effect of some of these unobservable factors remains constant for a given period (Miina, 1993), then it is possible to calibrate future increment by introducing into the model the stochastic effects predicted for a prior period; as a result, permits the calibration of growth models for a specific location and growth period (Calama and Montero, 2005; Lynch et al., 2005; Gutzwiller and Riffell, 2007).

Many regionally limited growth and yield models are available for this species (Ritchie, 1999). Since data collection and analysis procedures differ, comparisons of growth responses that may be due to geographic variation of the species are difficult. Data for these models are often compiled all or in part from temporary plots often using stem analysis techniques. Such data suffer from the same weaknesses as retrospective studies. The investigator is never certain that the response measured is the result of the stated condition (Uzoh and Oliver, 2006). Therefore, the objective of this study is to overcome these weaknesses by presenting a single distance-independent diameter increment model applicable throughout the range of ponderosa pine in the United States and by using only data from long-term permanent plots on sites capable of normal yields (Meyer, 1938).

## 2. Methods

### 2.1. The data base

The foundation of the data base is six levels-of-growing-stock studies established throughout the western United States in the 1960s. All used a common study plan that divided the range of ponderosa pine in the United States into five provinces and specified five or six stand density levels replicated three times (Myers, 1967). Results from individual installations have been reported previously (Table 1). These data were supplemented with initial spacing and other permanent-plot thinning studies. Individual tree data were from plots in planted stands or stands of natural origin and included a wide range of size classes (Tables 2 and 3). Stands were free or mostly free of competing shrubs that reduce growth of young ponderosa pine, especially in central Oregon and California (Oliver, 1984; Oliver and Edminster, 1988; Oliver, 1990; Cochran and Barrett, 1999). Trees in all plots in the data base were tagged allowing the collection of information on individual trees. The number of growing seasons between remeasurements was usually 5, but most plots were observed for a much longer period. Eighty-two percent of the plots were observed for 20 years or more-four 5-year growth periods. Basic records for each plot included latitude, elevation, aspect, slope percent, and plot size. Tree records at each remeasurement included diameter at breast height (dbh), and total height on a sample of trees. Diameter measurements were repeated on the same trees ensuring that the 5-year diameter increment is given by the difference between the two successive observations of diameter.

**Table 1**

Location and literature citations for five levels-of-growing-stock installations in ponderosa pine in western United States

Province	Installation name	Geographic location	Literature citation
I	Elliot Ranch	West slope northern Sierra Nevada, CA	Oliver (1997)
II	Lookout Mountain	East side of Cascade Range, OR	Cochran and Barrett (1999)
III	Crawford Creek	Blue Mountain, OR	Cochran and Barrett (1995)
IV	Black Hills	Black Hills, SD	Boldt and Van Deusen (1974)
V	Taylor Woods	Coconino Plateau, AZ	Ronco et al. (1985)

**Table 2**

Distribution of plots in each province by stand origin and tree size used to develop the 5-year periodic annual increment (PAI) in diameter model for managed even-aged stands of ponderosa pine throughout the western United States

	Province				
	I	II	III	IV	V
Number of plots					
Stand origin					
Natural	11	71	33	42	18
Planted	95	26	14	0	0
Stand size class					
Saplings	31	10	0	0	0
Poles	64	83	47	42	18
Sawtimber	11	4	0	0	0

### 3. Analysis

#### 3.1. The equation

Growth of the individual trees was potentially affected by four groups of variables: tree size and vigor effects, site effects, competitive effects, and regional effect. The combination of some of these predictor variables and the transformation of others were initially tested for predicting 5-year periodic annual increment (PAI) in diameter (cm) using multilevel mixed models analysis procedure. The multilevel mixed model included both fixed effects and random effects components. Between periods, between plots, between tree and within-tree differences were accounted for by including random effect parameters specific at those levels. The variable selection process involves a series of steps beginning with an initial data exploration that involves plotting the data and examining correlation statistics to identify those variables that may be useful in the model.

#### 3.2. Tree size effects

We started by defining the relationship between increment and size and accounting for the two different sizes of experimental

units: a spatial unit which is an individual tree and a set of temporal units which are the repeated measurements on individual trees. The following equation was used:

$$\ln(\text{PAIDBH}) = \beta_0 + \beta_1 \ln(\text{DBH}) + \beta_2 (\text{DBH}^2) + h_l + e_{j(l)} + e_{ik(jl)}, \quad (1)$$

where  $\ln(\text{PAIDBH})$  is the natural logarithm of 5-year periodic annual increment (PAI) in diameter at breast height (dbh) (cm);  $\ln(\text{DBH})$  is the value of the natural logarithm of initial dbh (cm);  $(\text{DBH}^2)$  is the value of the square of initial dbh (cm),  $\beta_0, \beta_1, \beta_2$  are regression coefficients,  $h_l$  is the random effect of the  $l$ -th location with  $h_l$  assumed to have an expected value of zero (0) and constant variance  $\sigma_{h_l}^2$ ,  $e_{j(l)}$  is a random error for plot  $j$  within locale  $l$  assumed to have an expected value of zero (0) and constant variance ( $\sigma_{P(L)}^2$ ), and  $e_{ik(jl)}$  is a random error for measurement  $k$  on tree  $i$  within plot  $j$  and locale  $l$  assumed to have an expected value of zero (0) and variance ( $\sigma^2$ ) with the covariance between observations  $k$  and  $k'$  on the same tree separated by  $d$  years following an autoregressive process:

$$\text{Cov}(e_{ik(jl)}, e_{ik'(jl)}) = \begin{cases} \sigma^2 \rho^{|d|}, & \text{if } i = i', k \neq k', j = j', l = l' \\ \sigma^2, & \text{if } i = i', k = k', j = j', l = l' \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

where  $\rho$  is the serial correlation coefficient for errors across time (5 years) on the same tree.

When the resulting predictive function is plotted against DBH, the resulting function is a skewed unimodal shape with a maximum between 20 and 30 cm (Fig. 1). Additionally, the intercept term,  $\beta_0$ , can be expanded to include other tree and site effects that modify diameter increment while still retaining the basic relationship between tree size and growth (Wykoff, 1990; Uzoh, 2001; Uzoh and Oliver, 2006).

#### 3.3. Site effects

For a model to adequately characterize tree growth, it must include some measure of site productivity (Spurr and Barnes, 1980). Latitude, longitude, aspect, slope, elevation, and site index

**Table 3**

Summary statistics for the data used to develop the 5-year periodic annual increment (PAI) in diameter model for managed even-aged stands of ponderosa pine throughout western United States

Variable	Number of trees observed	Mean	S.D.	Minimum	Maximum
HT (m)	34263	10.797	4.542	0.034	53.214
DBH (cm)	34263	18.167	8.801	0.254	98.044
SI (m)	34263	21.619	6.859	13.106	48.768
AGE (year)	34263	58.759	21.416	4.000	110.000
BAL (m <sup>2</sup> /ha)	34263	13.930	9.570	0	74.249
Plot variables	Number of plots	Mean	S.D.	Minimum	Maximum
ELEVA (m)	310	41.789	4.303	35.280	48.500
SLOPE (per)	310	6.466	7.264	0	42.000
ASPECT (rad)	310	116.951	100.942	0	360.000
SDI (trees/ha)	310	473.026	226.967	0	1444.220
LAT	310	42.312	4.104	35.28	48.5

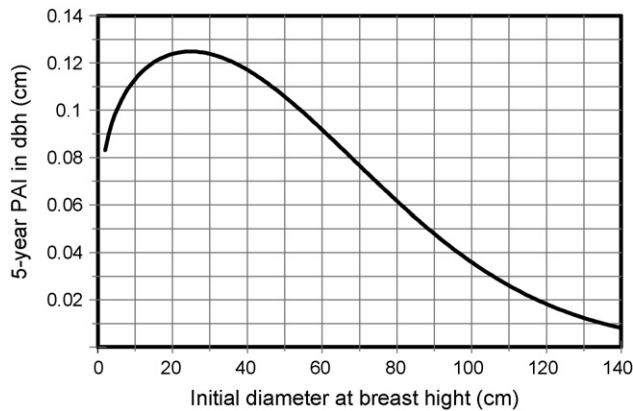


Fig. 1. Five-year periodic annual increment (PAI) in diameter by initial diameter at breast height (dbh) using the coefficients derived for Eq. (1).

were initially tested for site effects variables. The site quality indicator term is represented as:

$$\text{SITE} = \beta_3 \text{SI}, \quad (3)$$

where SI is site index (m) (Meyer, 1938). Meyer's site index was chosen because the data came from the widest geographic range of any site index system presently available. Provincial site index systems may more accurately portray sites within that province but they are widely divergent (Dunning and Reineke, 1933; Minor, 1964; Boldt and Van Deusen, 1974; Barrett, 1978; Powers and Oliver, 1978). Because, the main objective of this study is to provide forest managers with a single diameter increment model that can provide useful guidelines for a variety of management objectives throughout the geographic range of ponderosa pine, we had to use a range-wide site index system.

In addition to SI, other geoclimatic variation (OGV) may remain because of the range of variation in species characteristics and stand conditions in the study area, which extended from the Black Hills of South Dakota to the Pacific Coast. As a result, slope (SL), aspect (ASP), and latitude (LAT) terms can help in refining the overall site effect. These factors generally have no direct effect on tree growth, but act indirectly by influencing moisture, temperature, light, and other chemical and physical agents of the site. Slope and aspect are included using Stage's (1976) transformation. The combined effects of slope, aspect, and latitude are represented by OGV:

$$\text{OGV} = \beta_4 \text{SL}[\cos(\text{ASP})] + \beta_5 \text{LAT}, \quad (4)$$

where SL is the slope angle in percent, ASP is the aspect in radians, and LAT is latitude in degrees.

### 3.4. Competitive effects

Finally, the increment attained by an individual tree is also dependent on its competitive status relative to neighboring trees and the impact of management. Basal area in larger trees (BAL), stand density index (SDI), stand basal area (BA), and basal area in larger trees divided by dbh of the subject tree (BAL/DBH) were initially tested for competitive effects variables. The competitive effects (CE) term is represented by SDI (Reineke, 1933) for overall stand density and BAL/DBH for the individual tree's competitive position. SDI has a distinct advantage over stand basal area as a measure of stand density because it is less influenced by age and site quality. BAL has been used often as a tree-position variable in equations for predicting growth because it describes a tree's position in relation to all trees measured in a plot or stand (Ritchie

and Hann, 1985; Wyckoff, 1986; Dolph, 1988; Wyckoff, 1990; Uzoh et al., 1998; Uzoh, 2001; Uzoh and Oliver, 2006). The competitive effects (CE) term is represented by SDI and BAL/DBH:

$$\text{CE} = \beta_6 \text{SDI} + \beta_7 \frac{\text{BAL}}{\text{DBH}}, \quad (5)$$

where SDI is stand density index (T/ha), and BAL/DBH is the basal area in larger trees (m<sup>2</sup>/ha) divided by the dbh of the subject tree.

#### 3.4.1. Model selection

Many different tools can be used in evaluating competing models to determine which model is most appropriate. Most of these criteria are based on the presumption that you want to create a model that minimizes the unexplained variability (the mean squared error of prediction) with the fewest number of variables possible. Of the potential models, the one selected was chosen on the basis of the following criteria:

- The covariance structure was chosen among the two candidates of autoregressive errors and compound symmetry based on a maximum likelihood estimation of the fixed effects and random effects and choosing the structure that produced the smallest Akaike's Information Criterion (AIC) (Rawlings et al., 1998; Hastie et al., 2001; Burnham and Anderson, 2002).
- Restricted maximum likelihood (REML) was used to fit different fixed effects models. Then AIC was used to assess model fitness.
- Residual plots were examined to check on normality assumptions and the Spearman rank correlation coefficient was calculated for examining the stability of the variance across the range of independent variables (Carroll and Rupert, 1988).

AIC (The Fit Statistics) is defined as follows:  $\text{AIC} = -2 \log L + 2(p + 1)$ , where  $L$  is the maximum of the likelihood function and  $p$  is the number of predictors (including the intercept).

#### 3.4.2. Repeated measures analysis

Selecting an appropriate covariance model is important in repeated measures analyses. If an important correlation is ignored by using a model that is too simple, the risk of Type I error rates is increased for fixed effects tests. If the model is too complex, power and efficiency is sacrificed. This decision process can be assisted by using the goodness of model fit criteria (AIC).

In selecting covariance structure, we used the procedure outlined by Gutzwiller and Riffell (2007):

1. Fit the fixed effects portion of the model.
2. Identify a set of candidate covariance structures.
  - Consider ecological and biological characteristics of the dependent variables. For example, does the variance of the response variable fluctuate from year to year.
  - Consider parsimony of the covariance structure relative to available sample size. Many of the available covariance structures require a large number of extra parameters, which may exceed the number of parameters that can be confidently estimated for a given sample size.
3. Fit a separate mixed model (with an independent fixed effects portion) using each of the candidate covariance structures.
4. Select the most appropriate covariance structure using Akaike's Information Criteria (AIC). The covariance structure with the smallest value of the criteria is considered most desirable.

In this study based on the value of AIC, the autoregression covariance structure (with multiple observations on individual trees autocorrelated in time) outperformed other covariance



structures such as compound symmetry (with multiple observations on an individual tree equally correlated irrespective of time).

The random terms (a variance component for plots within locales and for trees within plots along with the correlation between successive measurements on individual trees) are represented, which provided the best relationship between 5-year periodic annual increment (PAI) in diameter (cm) of individual ponderosa pine trees, and tree size and vigor effects, site effects, competitive effects, and regional effect. There are several statistical packages that can estimate the coefficients of a linear regression model with random effects due to the repeated measures or other random blocking structures (Mixed General Linear Models (MGLM)). However, we used the SAS (v.9.1.3) software (SAS Institute, 2006) in our analysis because it is widely available, and it is the platform with which we are most familiar.

#### 4. Model testing and validation

Shugart (1984) defined model validation as “procedures, in which a model is tested on its agreement with a set of observations that are independent of those observations used to structure the model and estimate its parameters”. There are many types of validation methods available; some are qualitative and others are quantitative (Holmes, 1983; Sargent, 1999). The use of statistical tests in model validation has drawn much debate since the work of Wright (1972), because of the varying criteria for the “value” of models and the methods of determining it (Mayer et al., 1994; Morehead, 1996). Because each model is unique, no single validation technique or method is widely applied. For selecting the most suitable regression model, it is generally advisable to use some measure of lack of fit in combination with one or more test statistics (Kozak and Kozak, 2003). Therefore, it is important to know that model validation is not designed to prove that a model is correct (Popper, 1963), but rather to show that model predictions are close enough to independent data and that decisions made based on the model are defensible (Yang et al., 2004).

There are four procedures commonly used in model validation: (1) a comparison of predictions and coefficients with physical theory; (2) a comparison of results with those obtained by theory and simulation; (3) the use of new data; and (4) the use of data splitting or cross validation (Snee, 1977). Since a new data set is often not available, data splitting is regarded as an acceptable alternative by most practitioners provided that the data set is large enough (Yang et al., 2004).

The dataset was randomly split into 10 parts and 90% was used for initial model development and 10% was used for model validation. The final model was developed using the entire dataset. Using the validation dataset, the coefficients of the multilevel mixed model analysis and the regression model analysis were used to predict 5-year PAI in diameter (cm). The residuals were calculated by subtracting the predicted values from the observed values. Four lacks of fit statistics were used to check model accuracy:

$$\text{Mean Squared Error (MS Error)} = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2;$$

$$\text{Mean bias (MB)} = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i);$$

$$\text{Absolute Mean bias (A Mean bias)} = \frac{1}{n} \sum_{i=1}^n |(Y_i - \hat{Y}_i)|;$$

$$\text{and Mean Percent Error (Mean\% Error)} = \frac{1}{n} \sum_{i=1}^n \frac{(Y_i - \hat{Y}_i)}{Y_i};$$

where  $n$  is the number of observations;  $Y_i$  is an actual observation of a given dependent variable, and  $\hat{Y}_i$  is the actual predicted value

**Table 4**

Prediction mean squares error (MS Error, cm), mean bias (cm), absolute mean bias (A Mean bias, cm), and mean percent error (Mean %Error, cm) by mixed model and regression model using validation data set of size 10% of the total data set

Model	$n$	MS error	Mean bias	A mean bias	Mean %error
Mixed	15498	0.2797	−0.1835	0.3983	21
Regression	15498	0.2894	−0.2373	0.4084	24

of a given observation of the dependent variable. The validation results are presented in Table 4.

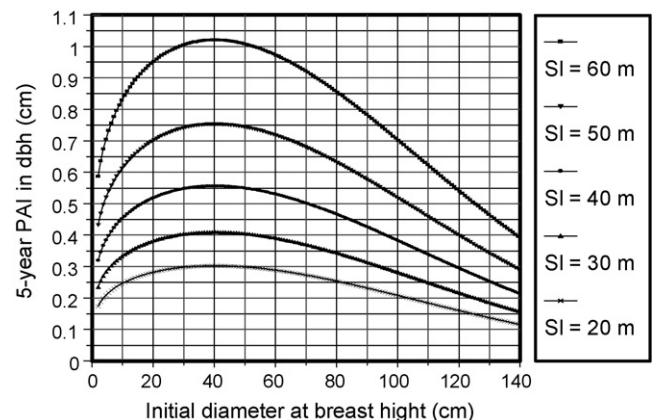
#### 5. Results

The following equation provided the best fit:

$$\begin{aligned} \ln(\text{PAIDBH}) = & b_0 + b_1 \ln(\text{DBH}) + b_2 \text{DBH}^2 + b_3 \text{SI} \\ & + b_4 \text{SL}[\text{COS}(\text{ASP})] + b_5 \text{LAT} + b_6 \text{SDI} + b_7 \frac{\text{BAL}}{\text{DBH}} \\ & + h_l + e_{j(l)} + e_{ik(jl)}, \end{aligned} \quad (6)$$

The random effects were trees, plots, and locations. Where  $\ln(\text{PAIDBH})$  is the natural logarithm of 5-year periodic annual increment (PAI) in diameter at breast height (dbh) (cm),  $\ln(\text{DBH})$  is the value of the natural logarithm of initial dbh (cm),  $\text{DBH}^2$  is the square of initial dbh (cm), SI is the Meyer's site index (m), SL is the average slope for the stand (percent), ASP is the average aspect for the stand (radians), LAT is the latitude for the stand in degrees, SDI is the stand density index (trees/ha), BAL/DBH is the basal area in larger trees ( $\text{m}^2/\text{ha}$ ) divided by the dbh of the subject tree (see Table 6),  $b_0, b_1, b_2, b_3, b_4, b_5, b_6$ , and  $b_7$ , are regression coefficients,  $h_l$  is the random effect of the  $l$ -th location with  $h_l$  assumed to have an expected value of zero (0) and constant variance  $\sigma_{h_l}^2$ ,  $e_{j(l)}$  is a random error for plot  $j$  within locale  $l$  assumed to have an expected value of zero (0) and constant variance ( $\sigma_{p(l)}^2$ ), and  $e_{ik(jl)}$  is a random error for measurement  $k$  on tree  $i$  within plot  $j$  and locale  $l$  assumed to have an expected value of zero (0) and variance ( $\sigma^2$ ) with the covariance between observations  $k$  and  $k'$  on the same tree separated by  $d$  years (see Eq. (2)).

The 5-year periodic annual increment (PAI) in diameter (cm) model developed [Eq. (6)] displayed a unimodal, positively skewed shape that is typical of tree growth processes (Fig. 2) (Assmann, 1970; Wyckoff, 1990; Uzoh, 2001; Uzoh and Oliver, 2006). The coefficients from Eq. (6), the variance components, and the



**Fig. 2.** Five-year periodic annual increment (PAI) in diameter by initial diameter at breast height (dbh) and site index (SI) for managed even-aged stands of ponderosa pine throughout western United States using the coefficients in Table 6 (a value of 40 was assigned for latitude, a value of 25 was assigned for both slope and aspect, a value of 120 was assigned for SDI, and a value of 20 was included for BAL/DBH).

autocorrelation coefficient are presented in Table 6. If the true regression coefficients were known, then the variance of new observation  $\hat{Y} = \ln(\text{PAIDBH})$  given the independent variables would be  $\sigma_L^2 + \sigma_{P(L)}^2 + \sigma^2$ . (Note that the serial correlation of the model is not involved in the variance of a single prediction. It would be necessary to use  $\rho$  if the variance was needed for some function of multiple measurements on a single tree).

The logarithmic-bias correction to the intercept term (Flewelling and Pienaar, 1981) was estimated by adding half of the mean squared error to the intercept term (Baskerville, 1972). Flewelling and Pienaar (1981) suggested that for degrees of freedom  $>30$  and  $S^2 < 0.5$ , the multiplicative correction of  $e^{S^2/2}$  is usually adequate. Since the residual mean squared error estimate is  $<0.5$  and the sample size is  $>30$ , Baskerville's correction should be a close approximation to the true logarithmic bias for the equation presented (Eq. (6)). Also, the residuals appeared to be normal. Therefore, Baskerville's method was used for the logarithmic-bias correction. All residual statistics presented in the tables and figures are based on transformed logarithmic values and the correction term is added into the intercept in Table 6.

## 6. Discussion

The relative importance of a variable is assessed by the change in size of the standard error of prediction without the variable in question. Table 5 shows the ranking of the variables based on this criterion. Within the fundamental constraint of site quality, tree position significantly influenced diameter growth. BAL/DBH ((basal area in larger trees ( $\text{m}^2/\text{ha}$ ))/(dbh of the subject tree)) had more effect on diameter growth than any other variable (Table 5). The increment attained by an individual tree is dependent on its competitive status relative to neighboring trees. Consequently, the coefficient of BAL/DBH is negative, indicating a competition modifier that would reduce diameter growth rates relative to a tree's competitive status. Therefore, the largest diameter tree in a plot would have a BAL value of zero, while the smallest diameter tree in the plot would have a BAL value near that of the plot's total basal area. As BAL decreases, the predicted increment increases. The more open-grown the tree, the less it is influenced by competitors because the measure of relative size is tied to stand density. As a result, dominance is less of a factor in increment predictions in sparsely stocked stands (Wykoff, 1990; Uzoh et al., 1998; Uzoh, 2001; Uzoh and Oliver, 2006).

Overall stand density as measured by SDI had the least influence on diameter growth (Table 5). The importance of SDI in the model suggests that diameter growth of all trees in a stand is affected by stand density—trees with the largest diameters as well as those with the smallest diameters. This relationship is in accordance with that reported for the two levels-of-growing-stock

installations in Oregon (Cochran and Barrett, 1995, 1999). Also, this confirms the findings of Hann and Hanus (2002).

After thinning from below in dense stands, BAL is unchanged, but predicted increment increases because SDI is lower. In rare instances when growth after thinning from above is modeled, predicted response may be overestimated, at least initially. Both BAL and SDI are reduced, causing a predicted growth increase greater than that for thinning from below. The response might be delayed until tree crowns and root systems of subordinate trees expand to take advantage of the added space. In general, however, the effects of BAL and SDI are biologically rational and simple in concept yet they can accommodate extensive variation in stand structure and site conditions (Wykoff, 1990; Uzoh et al., 1998; Uzoh and Oliver, 2006).

Site index (Meyer, 1938) (SI) had the second most effect on diameter growth than any other variable (Table 5). It is important to realize, however, that other factors were combined under the variable SI. The data were scattered over a vast geographic area of contrasting soils and climate, and included two varieties of ponderosa pine (*P. ponderosa* var. *ponderosa* and var. *scopulorum*). Some of the genetic differences may have affected 5-year periodic annual increment (PAI) in diameter. Nevertheless, what we called SI seemed to perform credibly in integrating and explaining these complex differences. A contributing reason for the good performance of site index may have been that stockability was not a problem. All data were from sites capable of the productivity estimated by Meyer (1938).

The natural logarithm of initial diameter ( $\ln(\text{DBH})$ ) had the fifth most effect on diameter growth. The size of initial diameter is an indication of a tree's competitive status within a plot or stand, and thus an expression of tree vigor; while the square of initial diameter ( $\text{DBH}^2$ ) had the fourth most effect on diameter growth (Table 5). The inclusion of  $\text{DBH}^2$  gave Eq. (6) its asymptotic approach to zero for large diameters, removing the need for imposition of an arbitrary maximum diameter (Fig. 2).

Stage's (1976) transformation of slope (SL) and aspect (ASP) ( $\text{SL}[\cos(\text{ASP})]$ ) had the sixth most effect on diameter growth (Table 5). The transformation of slope and aspect has two important properties, it is circular, and optima exist with respect to both slope and aspect. We tried Stage (1976) transformation of slope and aspect ( $(\text{SL}[\cos(\text{ASP})]$  and  $\text{SL}[\sin(\text{ASP})]$ ); however, only  $\text{SL}[\cos(\text{ASP})]$  was statistically significant. The *P*-values (2-tail) for  $\text{SL}[\sin(\text{ASP})]$  was 0.6091. Additionally, with the variable included in the model, AIC value increased by 10 points. As a result, we did not include the variable in our analysis. Also, we tried Stage and Salas (2007) transformation of elevation, aspect, and slope and that did not work either; consequently, we did not use their proposed transformation. Our plots were located at elevations where ponderosa pine was best suited at that latitude. As a result, at lower latitudes plots were located at higher elevations. Consequently, for Stage and Salas (2007) transformation of elevation, aspect, and slope to be successfully tested, we would have had to have an especially designed study that had plots at several elevations on the same aspect on both north and south slopes along a latitudinal transect.

Latitude had the third most effect on diameter growth (Table 5). Latitude is an important variable because more northerly locations tend to be cooler, with shorter growing seasons, than more southerly locations. Because diameter growth is sensitive to length of growing season, latitude, which is the model location coefficient, describes this climatic trend. Consequently, the coefficient of latitude is negative. These site factors (slope, aspect, and latitude) generally have no direct effect on tree growth, but act indirectly by influencing moisture, temperature, light, and other chemical and physical agents of the site (Uzoh, 2001). Since the factorial

**Table 5**  
Ranking of variables based on change in the size of the standard error of prediction without the variable in question for the mixed model analysis

Variable	Value of standard error of prediction without the variable in question	Value of standard error of prediction for the full model
BAL/DBH ( $\text{m}^2/\text{ha}$ )	0.624179	0.536395
SI	0.592815	
LAT	0.54632	
$\text{DBH}^2$	0.54554	
$\ln(\text{DBH})$	0.540287	
$\text{SL}[\cos(\text{ASP})]$	0.53877	
SDI	0.527001	

**Table 6**

Parameter estimates and variance components for the 5-year periodic annual increment (PAI) in diameter model for managed even-aged stands of ponderosa pine throughout western United States using the SAS system MIXED model analysis procedure

Parameter	Estimate	Standard error	P-values (2-tail)	Lower	Upper
Intercept	0.50994513	0.858436	0.6713	−1.3506	2.08281
ln(DBH) (cm)	0.508652508	0.004251	0.0001	0.500322	0.51698
DBH <sup>2</sup> (cm)	−0.000068368	0.0000041	0.0001	−0.00008	−0.00006
SI (m)	0.030328112	0.004759	0.0001	0.02092	0.03974
SDI (trees/ha)	−0.001152023	0.000013	0.0001	−0.00118	−0.00113
BAL/DBH (m <sup>2</sup> /ha)	−0.016249869	0.000155	0.0001	−0.01655	−0.01595
LAT	−0.063794567	0.019274	0.0017	−0.10244	−0.02514
SLCOSASP	0.004339865	0.002149	0.0443	0.000111	0.00856
Variance component					
$\sigma^2_{\text{Plot(Locale)}}$	0.027326607	0.002495	0.0001	0.023027	0.03296
$\sigma^2_{\text{Locale}}$	0.083486458	0.018677	0.0001	0.056266	0.13671
$\rho_{\text{Serial correlation coefficient}}$	0.79274766	0.051365	0.0001	0.692075	0.89342
$\sigma^2_{\text{Residual}}$	0.176865694	0.000638	0.0001	0.175621	0.17812

approach to site quality evaluation is based on relatively stable features of the environment and not on current vegetation characteristics, it can be applied to both disturbed and undisturbed sites. All parameter estimates were logical and in line with expectations (Table 6).

PROC MIXED will not calculate the coefficients of variable with very high degrees of multicollinearity (SAS, 2002). Consequently, elevation was dropped in favor of latitude which is an obvious location variable, because a high degree of multicollinearity existed between the two variables, and because latitude had more effect on diameter growth than did elevation. The two variables were confounded because ponderosa pine is found at increasing elevations as latitudes decrease. Age and longitude were dropped because they were highly correlated with the more important variable, site index. Age is usually correlated with SI in managed stands because the range of tree sizes is usually restricted to saplings through small sawtimber. The bulk of the plots in our study were in pole size stands (see Table 2). Poles on poor sites tend to be older than do poles on good sites. And longitude was correlated with SI because our data spanned the geographic range of ponderosa pine from South Dakota to the Pacific Coast. Few if any site east of the Sierra Nevada or Cascade Range can equal the better sites on the Pacific slope to the West.

One of the major obstacles we encountered in trying to use the regression analysis approach to analyze the data set was the need to account for the more complex nature of the covariance structure of the model because of the multiple sizes of plots and the correlation over time from measuring the same trees. We found that there were statistically significant variance components for plots within locales and for locales (Table 6). Therefore, we needed to account appropriately for the error structure imposed by the sampling scheme, and multilevel linear mixed model approach accomplishes that successfully. What makes the multilevel linear mixed models procedure unique in modeling repeated measured data with hierarchical structure is the ability of the models to include both fixed regression parameters (fixed effects) that describe the shape of the typical growth curve over the entire population, and random regression coefficients (random effects) that individualize the curve to capture site-specific, tree-specific, or other unit-specific characteristics of the growth pattern. The random effect included in the model corresponds to sampling units (locale, plots, and trees) that have a hierarchical structure with multiple levels.

The model developed in this analysis appears to be well behaved. The model is enhanced by confining the data set to permanent plots in pure even-aged stands of ponderosa pine and following the growth of individually tagged trees for long time

periods. Eighty-two percent of the plots were followed for 20 years or longer. The diverse ecological requirements of ponderosa pine trees represented in the data base should enhance model performance and would encourage use of the model throughout the range of ponderosa pine in the United States.

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